WAVEFORM–BALANCE METHOD FOR NONLINEAR MESFET AMPLIFIER SIMULATION

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ABSTRACT

A new algorithm for the analysis of a nonlinear MESFET amplifier is presented. The algorithm is a hybrid method which uses both time and frequency domain analysis. However, unlike the Harmonic Balance method, the solution is optimized in the time domain where the EXACT error gradient matrix is calculated. The new method is shown to have excellent convergence speed and accuracy.

INTRODUCTION

For the first time, an efficient algorithm (1), which we call the Waveform-Balance method, is used for the analysis of the nonlinear MESFET amplifier. The method is different from the well known Harmonic Balance method. Here, the steady-state solution is sampled and optimized in the time domain rather than in the frequency domain. In the new method, the EXACT error gradient matrix required for the optimization procedure is calculated in the time domain. Whereas, in the Harmonic Balance method, the error gradient matrix can only be calculated numerically in the frequency domain by comparing the error function values of two consecutive iterations. The new method is shown in this work to have good accuracy and convergence rate.

PROBLEM FORMULATION

The Waveform-Balance algorithm is a hybrid method which uses both time and frequency domain analysis. In the Waveform-Balance method, the amplifier circuit is first decomposed into linear and nonlinear subcircuits. The steady-state voltages at the nodes connecting the linear and nonlinear subcircuits are first sampled in the time domain. The voltage amplitudes at each sampling point is optimized using an iteration scheme to satisfy the current balance condition. In the iteration procedure, the error gradient matrix in the time domain is calculated EXACTLY.

To understand the Waveform Balance algorithm, consider a nonlinear amplifier circuit shown in Figure 1. The nonlinear

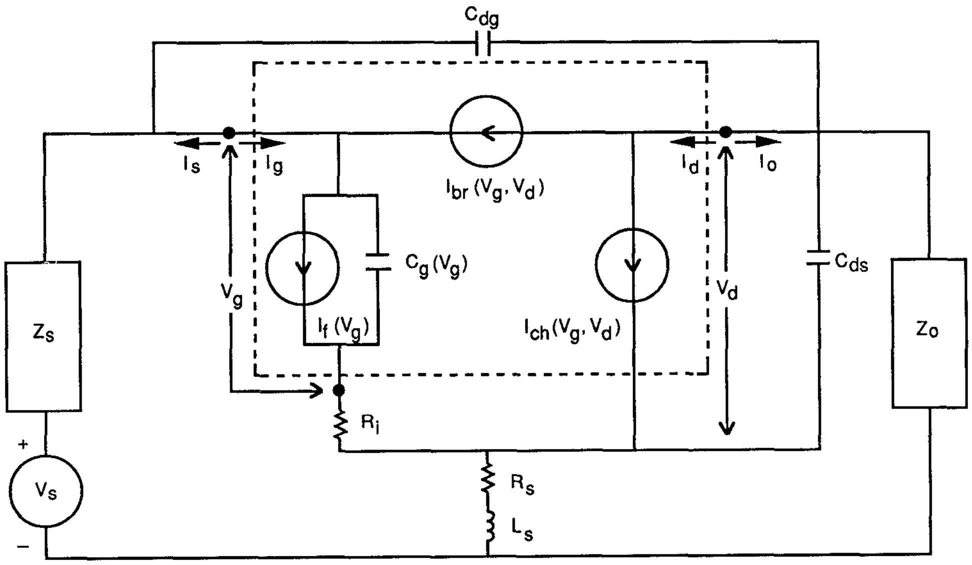

Figure 1 Nonlinear MESFET amplifier circuit.

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subcircuit is enclosed by dotted lines in Figure 1. The nonlinear elements are $I_f(V_g)$, $C_g(V_g)$, $I_{ch}(V_g,V_d)$, and $I_{br}(V_g,V_d)$, which are represented by explicit nonlinear expressions. The exact expressions of these elements are not important for the following discussions.

We are seeking the steady-state voltages, V_g and V_d, at the nodes connecting the linear and nonlinear subcircuits that minimize the 2N error functions,

$$F_i = Is_i + Ig_i$$
$$F_{N+1} = Io_i + Id_i \qquad \text{for } i = 1,2,\ldots,N \qquad (1)$$

where N is the number of sampling points per period of the fundamental frequency. Is_i is the amplitude of the current $I_{s(t)}$ (shown in Figure 1) evaluated at each sampling point. Ig_i, Io_i, and Id_i are interpreted in the same manner.

ITERATION SCHEME

Figure 2 is the flow chart of the Waveform Balance algorithm. The algorithm starts with an initial guess of Vg_i and Vd_i (Vg_i and Vd_i are the amplitudes of $V_{g(t)}$ and $V_{d(t)}$ at each sampling point). Currents (at each sampling point) flowing into the nonlinear subcircuit are then calculated as

$$Ig_i = If(Vg_i) + Cg(Vg_i) \cdot Vg_i' - Ibr(Vg_i, Vd_i)$$
$$Id_i = Ich(Vg_i, Vd_i) + Ibr(Vg_i, Vd_i) \text{ for } i = 1,2,\ldots,N \quad (2)$$

where Vg_i' is the time derivative of V_g at each sampling point. To calculate the currents flowing into the linear subcircuit (Is_i and Io_i), the voltages V_g and V_d are first discrete-Fourier transformed (FFT) to the frequency domain, then applied to the linear subcircuit. The resulting currents are then calculated by linear circuit analysis. They are subsequently converted back to the time domain by inverse-Fourier transform (IFT).

Next, the error functions are calculated using Eq. (1). In each iteration, the time domain sampled voltages, Vg_i and Vd_i, are updated via

$$[V]_k = [V]_{k-1} - 1/R \cdot [G]^{-1} * [F_i]$$
$$= [V]_{k-1} - 1/R \cdot [\delta V] \qquad (3)$$

where

$$[V] = [Vg_1, Vg_2, \ldots, Vg_N, Vd_1, Vd_2, \ldots, Vd_N]',$$
$$[F_i] = [F_1, F_2, \ldots, F_{2N}]',$$

subscript k is the kth iteration, and R is the relaxation factor. [G] is the 2N x 2N error gradient matrix, which will be obtained by the method described next.

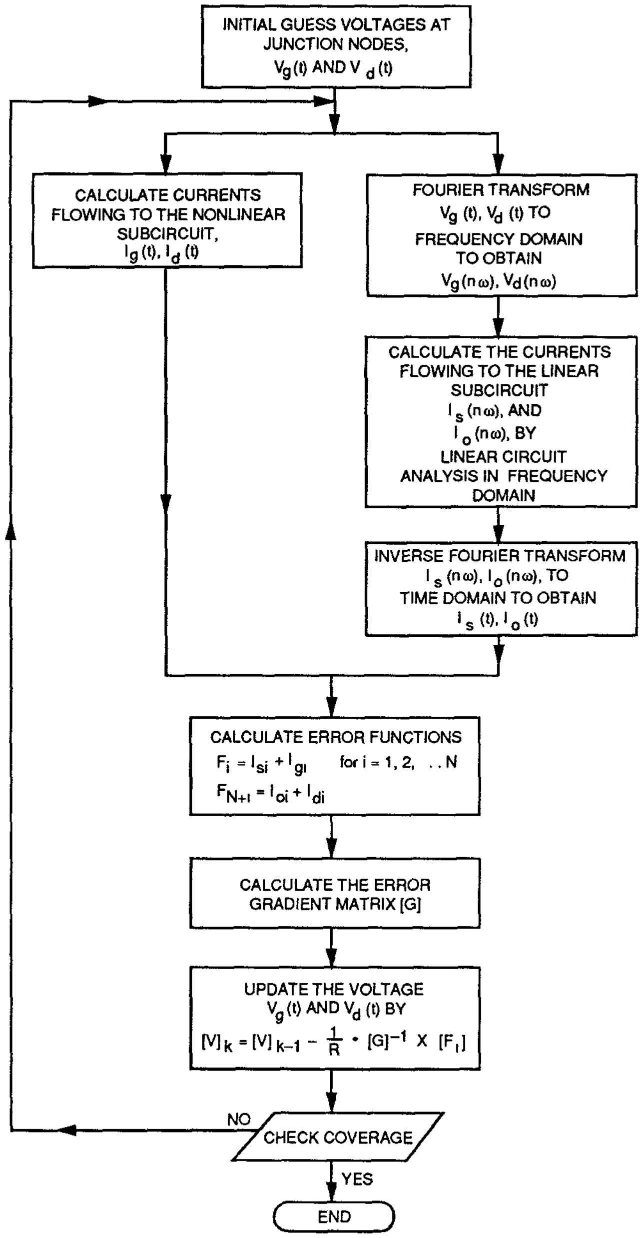

Figure 2 Flow chart of Waveform Balance Method.

ERROR GRADIENT MATRIX CALCULATION

The error gradient matrix [G] can be written as $[G] = [G_e] + [G_n]$, where $[G_e]$ and $[G_n]$ are the error gradient matrices contributed by the linear and nonlinear subcircuits, respectively. These matrices can be decomposed into submatrices as shown below

$$[G_n] = \begin{bmatrix} [\partial Ig/\partial Vg] & [\partial Ig/\partial Vd] \\ [\partial Id/\partial Vg] & [\partial Id/\partial Vd] \end{bmatrix} \quad [G_e] = \begin{bmatrix} [\partial Is/\partial Vg] & [\partial Is/\partial Vd] \\ [\partial Io/\partial Vg] & [\partial Io/\partial Vd] \end{bmatrix} \quad (4)$$

To see how $[G_n]$ and $[G_e]$ are calculated, consider first $[G_n]$, which is the error gradient matrix due to the nonlinear subcircuit. This matrix can be evaluated exactly. For example, the (i, j) element of the submatrix $[\partial Id/\partial Vd]$ is calculated as follows:

$$\partial Id_i/\partial Vd_j = \partial Ich(Vg_i, Vd_i)/\partial Vd_j + \partial Ibr(Vg_i, Vd_i)/\partial Vd_j \quad (5)$$

The other three submatrices of $[G_n]$ are calculated using the same principle.

To calculate $[G_e]$ (the error gradient matrix due to the linear subcircuit), consider the submatrix $[\partial Is/\partial Vg]$ of $[G_e]$ as an example

$$[\partial Is/\partial Vg] = \begin{bmatrix} \partial Is_1/\partial Vg_1 & \partial Is_1/\partial Vg_2 \ldots\ldots\ldots \partial Is_1/\partial Vg_N \\ \partial Is_2/\partial Vg_1 & \ldots\ldots\ldots \\ \cdot & \ldots\ldots\ldots \\ \cdot & \ldots\ldots\ldots \\ \partial Is_N/\partial Vg_1 & \ldots\ldots\ldots \end{bmatrix} \quad (6)$$

The first column of $[\partial Is/\partial Vg]$ the ratio of the perturbation of I_s at each sampling point with respect to the perturbation of V_g at the first sampling point. To calculate these ratios, the time domain series $\{\partial Vg_1, 0, 0, \ldots\}$ of N points,

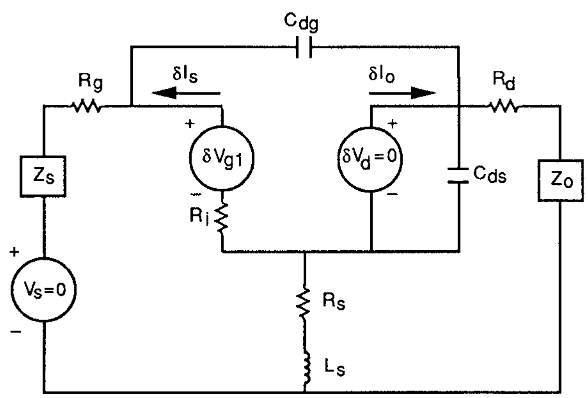

Figure 3 Circuit topology for calculating $[\partial I_s/\partial V_g]$.

is converted to the frequency domain by FFT. This perturbation is then applied to the linear subcircuit as shown in Figure 3, and the resulting current is transformed back to the time domain as $\{\partial Is_1, \partial Is_2, \ldots, \partial Is_N\}$. This series is equal to $\{\partial Is_1/\partial Vg_1, \partial Is_2/\partial Vg_1, \ldots \partial Is_N/\partial Vg_1\}$ by making $\partial Vg_1 = 1$. The second column of $[\partial Is/\partial Vg]$ is the ratios of perturbation of I_s with respect to the perturbation of Vg at the second sampling point. It can be shown that the second column is related to the first column via

$$\partial Is_{i+1}/\partial Vg_2 = \partial Is_i/\partial Vg_1 \quad \text{for } i = 1,2,3,\ldots, N-1$$

$$\partial Is_1/\partial Vg_2 = \partial Is_N/\partial Vg_1 \quad when\ i = N \quad (7)$$

The remaining columns of $[\partial Is/\partial Vg]$ are obtained by shifting the first column accordingly. The three other submatrices of $[G_e]$ can be constructed by the same method. It can be shown that $[G_e]$ does not change from iteration to iteration. Therefore, it needs to be calculated only once.

VERIFICATION

The new algorithm is validated by comparing its simulation results of a nonlinear amplifier circuit with the results generated by the commercial software LIBRA. Figure 4 shows the simulated output power versus input power curves for two different load impedances. As can be seen from Figure 4, the results produced by the Waveform-Balance and LIBRA are very close. When generating these data using Waveform-Balance, the input power is swept from 5 to 40 mW

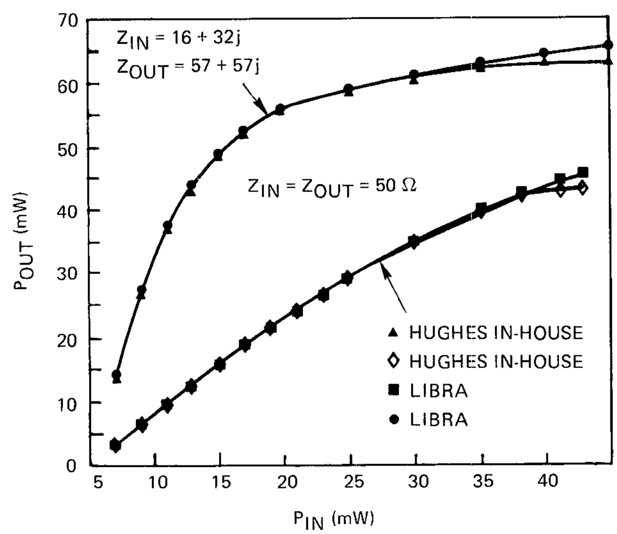

Figure 4 P_{out} versus P_{in} curves of a MESFET simulated at 10 GHz.

in 5-mW steps. On the average, it takes less than four iterations to calculate each data point, and each iteration takes about 0.25 seconds of CPU time on a VAX-11/780 computer.

CONCLUSION

The Waveform-Balance algorithm presented here is shown to be an accurate and efficient algorithm for nonlinear amplifier simulation. This algorithm can also be applied to other varieties of nonlinear circuits.

REFERENCE

(1) P.W. Van Der Walt, "Efficient technique for solving nonlinear mixer pumping problem," *Elec. Lett.*, Vol. 21, pp. 899-900, Sept. 1985.